Algebra Imhoff
10/15/13

15. Find the equation of the normal line to the curve $xy + y^2 - 2x = 0$ at the point $(1, -2)$

$[x\frac{dy}{dx} + y] + 2y\frac{dy}{dx} - 2 = 0$

$\frac{dy}{dx}(x + 2y) = 2 - y$

$\frac{dy}{dx} = \frac{2-y}{x+2y}$

$= \frac{2-(-2)}{1+2(-2)}$

$m_1 = \frac{4}{-3}$

$m_2 = \frac{3}{4}$

$y + 2 = \frac{3}{4}(x - 1)$

$y = \frac{3}{4}x - \frac{11}{4}$

16. Given $g(2) = 3$ $g'(2) = -2$ $h(x) = -1$ $h'(2) = 4$ find $f'(2)$ *if* $f(x) = g(x)h(x)$

$f'(2) = g(x)h'(x) + h(x)g'(x)$

$f'(2) = 3(4) + (-1)(-2)$

$f'(2) = 12 + 2$

$f'(2) = 14$

17. $f(x) = \begin{cases} x^2 + 2, & x \geq 1 \\ 2x + 1, & x < 1 \end{cases}$ Find $f'(x) = \begin{cases} 2x \\ 2 \end{cases}$ then find $f'(1) = 2$

18. The surface area of a sphere of radius r is given by $A = 4\pi r^2$. Find the rate of change of A with respect to r.

$\frac{dA}{dr}$ $\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$

$A = 4\pi r^2$

$\frac{dA}{dr} = 8\pi r \frac{dr}{dr}$

$\frac{dA}{dr} = 8\pi r$

I did not find the rate with respect to r, instead I did it! so I did $\frac{dr}{dt}$ instead of $\frac{dr}{dr}$ which would have cancelled out because it's 1. This was an error on reading the problem thoroughly

19. The circumference of a circle is increasing at a rate of $\frac{3\pi}{5}$ inches per minute. When the circumference is 6π inches, how fast is the area of the circle increasing?

$C = 2\pi r$

$6\pi = 2\pi r$

$\frac{6\pi}{2\pi} = r$

$r = 3$

$C = \pi r^2$

$6\pi = \pi r^2$

$\frac{6\pi}{\pi} = r^2$

$6 = r^2$

$r = \sqrt{6}$

$C = 2\pi r$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt} \cdot r(0)$

$\frac{dC}{dt} = 2\pi \frac{dr}{dt}$

$\frac{3\pi}{5} = 2\pi \frac{dr}{dt}$

$\frac{3}{5\pi} \cdot \frac{1}{2\pi} = \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{3}{10\pi}$

$6\pi\left(\frac{2\pi}{5}\right) = 2\pi(\sqrt{6})\frac{dr}{dt}$

$\frac{18}{5}\pi^2 = 2\pi\sqrt{6}\frac{dr}{dt}$

$\frac{dr}{dt} = \frac{18/5\,\pi^2}{2\pi\sqrt{6}}$

$\frac{dr}{dt} = \frac{18}{5} \cdot \frac{1}{2\sqrt{6}}(\pi)$

$\frac{dr}{dt} = \frac{18\pi}{10\sqrt{6}}$

$\frac{dr}{dt} = \frac{9\pi}{5\sqrt{6}}$ inches/minute

$C = 2\pi r$

$6\pi = 2\pi r$

$\frac{6\pi}{2\pi} = r$

$r = 3$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi(3)\left(\frac{3}{10}\right)$

$\frac{dA}{dt} = 2\pi\frac{9}{10}$

$\frac{dA}{dt} = \frac{18\pi}{10}$

$\frac{dA}{dt} = \frac{9\pi}{5}$

$\frac{dA}{dt} =$

$C = 2\pi r$

$A = \pi r^2$

$C = \pi r^2$

$C' = 2\pi r \frac{dr}{dt}$

I used the wrong equation ($C = \pi r^2$) to find r when I should have used $C = 2\pi r$ to find $\frac{dr}{dt}$. and $C = 2\pi r$ to find R. After I found $\frac{dr}{dt}$ and R I had to plug them into my derivative equation: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$ to find the rate of area.